Name: _______________ Date: _______________

Answer Key: Variable Acceleration & Vector Calculus Physics Challenge for University

Examine projectile motion in resistive media and non-linear paths across 10 rigorous analytical problems requiring calculus-based synthesis.

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1. A particle moves along a trajectory defined by r(t) = (acosωt)i + (bsinωt)j. At any time t, the acceleration vector a(t) is:

Answer: B) Directed toward the origin and proportional to the displacement r(t).

Differentiating r(t) twice yields $a(t) = -\omega^2 r(t)$, which signifies that acceleration is always anti-parallel to the displacement vector, directed toward the origin (simple harmonic or elliptical motion).

2. An object experiences a jerk (the time derivative of acceleration) that is constant and non-zero. The position of this object as a function of time is best described by a polynomial of degree ____.

Answer: B) Three

If the third derivative of position (jerk) is constant, integrating thrice with respect to time yields a cubic function (t^3) for the position.

3. In a curvilinear path, it is possible for an object to have a constant speed and a non-zero acceleration simultaneously.

Answer: A) True

In uniform circular motion, the speed is constant, but the direction of velocity changes continuously, resulting in a centripetal acceleration vector.

4. Consider a rocket whose acceleration increases linearly with time: a(t) = kt. If the rocket starts from rest at the origin, what is its displacement x after time T?

Answer: C) $(1/6)kT^3$

Integrating a(t)=kt gives $v(t)=1/2kt^2$. Integrating v(t) gives $x(t)=1/6kt^3$.

5. A projectile is launched at an angle θ with initial velocity v. In the presence of linear air resistance (F = -bv), the time taken to reach maximum height is _______ than in a vacuum.

Answer: D) Shorter

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Air resistance acts in the same direction as gravity during the ascent, increasing the magnitude of the downward acceleration and thus reducing the time to reach zero vertical velocity.

6. A particle moves such that its velocity is given by v = kx, where k is a constant. The acceleration of the particle is:

Answer: B) $k^2 / 2$

Using a = v(dv/dx), we get a = (kx) * (k / 2x) = $k^2/2$. This represents constant acceleration.

7. The area under an acceleration-time graph from t1 to t2 represents the total displacement of the object during that interval.

Answer: B) False

The integral of acceleration with respect to time represents the change in velocity (v), not the displacement (x).

8. A ball is thrown vertically upward in a medium where resistance is proportional to the square of the speed (v^2). Which statement regarding the terminal velocity (Vt) is correct?

Answer: A) It is reached when the drag force equals the weight of the ball.

Terminal velocity is defined as the constant speed reached when the sum of the drag force and buoyancy equals the downward force of gravity, resulting in zero net acceleration.

9. If a particle's position vector is r(t) = t^2 i + e^t j, the tangential component of acceleration at t=0 is ________.

Answer: B) 1

v(t) = 2t i + e^t j; at t=0, v = (0,1). a(t) = 2 i + e^t j; at t=0, a = (2,1). The tangential acceleration is the projection of 'a' onto 'v': (a·v)/|v| = (0*2 + 1*1)/1 = 1.

10. In 2D kinematics, if the x-component of acceleration is zero, the horizontal velocity of the particle must remain constant regardless of the y-component of acceleration.

Answer: A) True

In orthogonal coordinate systems, the components of motion are independent. If ax = 0, then dvx/dt = 0, meaning vx is constant.