

Name: _____ Date: _____

Answer Key: Escape Velocity or Bust: Kinetic Mastery for College Physics

Calculate orbital perturbations and non-linear trajectories to ensure your deep-space probe doesn't drift into the cosmic void.

1. A particle moves along a path such that its position vector is given by $r(t) = (A\cos(\omega t))i + (A\sin(\omega t))j$. Which of the following best describes the relationship between the particle's velocity and acceleration vectors?

Answer: C) They are mutually perpendicular at all times.

In uniform circular motion, the velocity vector is tangential to the path while the acceleration (centripetal) is directed toward the center. Their dot product $v \cdot a$ is zero, confirming they are perpendicular.

2. In a frame of reference undergoing constant linear acceleration, the kinematic equations of motion for a projectile remain invariant compared to an inertial frame.

Answer: B) False

In a non-inertial accelerating frame, fictitious forces (pseudoforces) must be introduced, altering the effective acceleration vector and thus the coordinate-based kinematic equations.

3. The time-dependent acceleration of a research submersible is defined by $a(v) = -kv^2$, where k is a constant and v is velocity. If the initial velocity is v_0 at $t=0$, the velocity as a function of time is $v(t) =$ _____.

Answer: A) $v_0 / (1 + v_0 kt)$

Applying the separation of variables to $dv/dt = -kv^2$ yields $v^{-2}dv = -k dt$. Integrating results in $-1/v = -kt + C$. Using the initial condition $C = -1/v_0$ leads to $v(t) = v_0 / (1 + v_0 kt)$.

4. Consider a projectile launched with velocity ' v ' at an angle ' θ ' on an inclined plane that itself makes an angle ' ϕ ' with the horizontal. To maximize the range ' R ' along the incline, what must be the relationship between θ and ϕ ?

Answer: C) $\theta = 45^\circ + \phi/2$

Using the range formula for an inclined plane and differentiating with respect to the launch angle, the maximum range is achieved when the launch angle bisects the angle between the vertical and the incline: $\theta = (90 + \phi) / 2 = 45 + \phi/2$.

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5. The jerk of an object, defined as the third derivative of displacement with respect to time, must be zero for the standard kinematic equations (e.g., $\Delta x = v_0t + \frac{1}{2}at^2$) to be valid.

Answer: A) True

The standard SUVAT equations assume constant acceleration. Since jerk is the rate of change of acceleration (da/dt), a non-zero jerk implies acceleration is not constant, rendering those specific formulas invalid.

6. A particle moves such that its velocity is $v = ks$, where s is the distance covered and k is a positive constant. The acceleration of the particle is _____.

Answer: A) $k^2/2$

Using the chain rule $a = v(dv/ds)$, we have $v = ks^{1/2}$ and $dv/ds = (k/2)s^{-1/2}$. Multiplying them: $a = (ks^{1/2}) * (k/2 * s^{-1/2}) = k^2/2$.

7. In polar coordinates (r, θ) , the radial component of acceleration for a particle moving in a plane is given by which expression?

Answer: B) $d^2r/dt^2 - r(d\theta/dt)^2$

In curvilinear motion, the radial acceleration contains the linear radial acceleration term and the centripetal term (subtractive) arising from the change in direction of the radial unit vector.

8. If the velocity of a particle is zero at a given instant, the acceleration of that particle at that same instant must also be zero.

Answer: B) False

A classic counterexample is a ball thrown vertically; at the peak of its trajectory, $v=0$, but acceleration is a constant -9.8 m/s^2 .

9. A relativistic particle moves with a velocity fraction ' β ' of the speed of light. If its position is measured in frame S , how does the observed velocity change when transforming to frame S' moving at velocity ' v '? This requires the use of:

Answer: B) Lorentz Velocity Addition

Galilean relativity fails at high speeds; the Lorentz transformation for velocities must be used to ensure that the speed of light remains invariant across all inertial frames.

10. A point moves along the curve $y = x^2/2$ with a constant speed ' v '. The magnitude of its acceleration at the origin $(0,0)$ is _____.

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Answer: A) v^2

At the origin, the radius of curvature $\rho = [1 + (dy/dx)^2]^{3/2} / |d^2y/dx^2|$. For $y = x^2/2$, $dy/dx = x$ (0 at origin) and $d^2y/dx^2 = 1$. Thus $\rho = 1$. Normal acceleration $a = v^2/\rho = v^2/1 = v^2$.